TRINITY COLLEGE

**Semester One Examination, 2022**

**Question/Answer booklet**

# MATHEMATICS METHODS UNIT 3

If required by your examination administrator, please place your student identification label in this box

## Section Two: Calculator-assumed

WA student number: In figures | | | | | | | | |

In words _____

_____

Your name _____

## Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): | |

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet
Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---|---|---|---|---|---|
| Section One: Calculator-free | 7 | 7 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 98 | 65 |
| | | | | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
   Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

| Markers use only | | |
|---|---|---|
| Question | Maximum | Mark |
| 8 | 7 | |
| 9 | 8 | |
| 10 | 8 | |
| 11 | 8 | |
| 12 | 8 | |
| 13 | 8 | |
| 14 | 9 | |
| 15 | 7 | |
| 16 | 8 | |
| 17 | 10 | |
| 18 | 10 | |
| 19 | 7 | |
| S2 Total | 98 | |
| S2 Wt (×0.6633) | 65% | |



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**Section Two: Calculator-assumed** **65% (98 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

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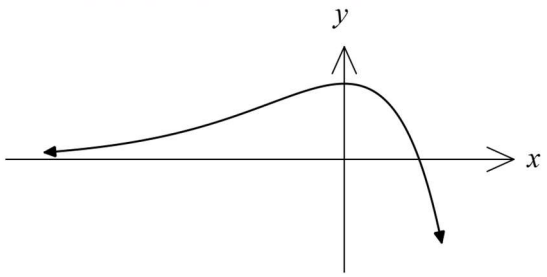
**Question 8** **(7 marks)**

Let $f(x) = (2 - x)e^{0.5x}$.

The graph of $y = f(x)$ is shown at right.




Use calculus to determine the coordinates of the stationary point, the point of inflection and to justify that the stationary point is a local maximum.



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**Question 9** **(8 marks)**

A small body moving in a straight line has an initial velocity of $15$ cm/s as it leaves point $P$. The acceleration of the body at time $t$ seconds is $6 - 1.5t$ cm$^2$/s, $t \geq 0$.

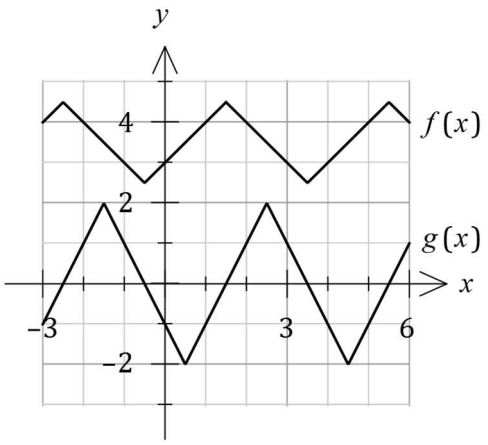
(a) Determine the displacement of the body relative to $P$ after $2$ seconds. (4 marks)

(b) Determine the maximum velocity of the body. (2 marks)

(c) Determine the maximum displacement of the body relative to $P$. (2 marks)

**Question 10** **(8 marks)**

The graphs of the continuous functions $y = f(x)$ and $y = g(x)$ are shown at right.




(a) Evaluate the derivative of $f(x)g(x)$ at $x = -2$. (2 marks)

(b) Evaluate the derivative of $f\big(g(x)\big)$ at $x = 5$. (3 marks)

(c) Evaluate the derivative of $\dfrac{g'(x)}{f(x)}$ at $x = 0$. (3 marks)

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**Question 11** **(8 marks)**

Due to the natural variability in the size, density and so on of fruit and syrup used in canning peaches, a cannery purposely overfills the cans it produces. However, some cans still end up being underweight and the probability $p$ that a canning machine produces such a can is known to be constant. Random samples of $n$ cans are taken from the machine and the number $X$ of underweight cans in each sample is recorded. The mean and standard deviation of $X$ are $2.16$ and $1.44$ respectively.

(a) $X$ is a discrete random variable. Explain why it is **discrete** and **random**. (2 marks)

(b) Name the distribution of $X$ and determine the value of $n$ and the value of $p$. (3 marks)

(c) Another canning machine produces an underweight can with a probability of $0.015$. Determine the probability that when a random sample of $30$ cans from this machine are weighed

(i) exactly one of the cans is underweight. (2 marks)

(ii) more than one of the cans are underweight. (1 mark)





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**Question 12** **(8 marks)**

A bag contains four red and three blue balls. Two balls are drawn at random and in succession from the bag. At each draw, if the ball is red it is replaced in the bag, and otherwise the ball is not replaced. Let $X$ be the number of red balls drawn.

(a) Determine $P(X = 2)$. (1 mark)

(b) Use exact values to complete the probability distribution table for $X$ below. (3 marks)

| $x$ | $P(X = x)$ |
|---|---|
| 0 | |
| 1 | |
| 2 | |

(c) Determine the mean and variance of $X$. (4 marks)

**Question 13** **(8 marks)**

The following table shows the probability distribution of a discrete random variable $X$, where $k$ is a constant.

| $x$ | $-2$ | $0$ | $1$ | $3$ |
|---|---|---|---|---|
| $P(X = x)$ | $4k^2$ | $0.15$ | $2k$ | $0.1$ |

(a) Determine the value of $k$. (3 marks)

(b) Determine $E(X)$. (2 marks)

(c) Given that $\text{Var}(X) = 2.31$, determine the following for the discrete random variable $Z$:

(i) $E(Z)$ when $Z = 5X - 3$. (1 mark)

(ii) $\text{Var}(Z)$ when $Z = \frac{X}{3} + 2$. (1 mark)

(iii) The standard deviation of $Z$ when $Z = 5(2 - X)$. (1 mark)





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**Question 14** **(9 marks)**

A full water tank takes $31$ seconds to empty. The volume $V$ litres of water in the tank, $t$ seconds after emptying began, is changing at a rate given by

$$\frac{dV}{dt} = \sqrt[3]{4t+1} - 5, \qquad 0 \leq t \leq 31.$$

(a) Determine the initial rate of change of volume. (1 mark)

(b) Use the increments formula to estimate the volume of water that empties from the tank during the first one-tenth of a second. (2 marks)

(c) Determine the initial volume of water in the tank. (3 marks)



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(d) Determine the time, to the nearest $0.01$ second, when the tank is half full. (3 marks)





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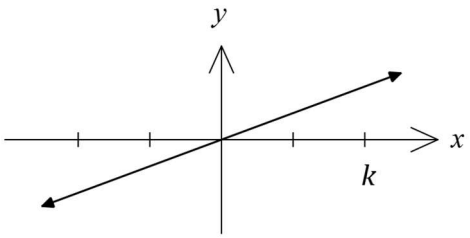
**Question 15** **(7 marks)**

(a) Consider the function $f(x) = mx$, where $m$ is a constant.
The graph of $y = f(x)$ is shown at right, $k$ is a constant and

$$\int_0^k f(x)\,dx = 4.$$




Determine the value of

(i) $\int_{-k}^{k} f(x)\,dx.$ (1 mark)

(ii) $\int_{-k}^{0} 2f(x+k)\,dx.$ (2 marks)

(b) The polynomial function $g(x)$ is such that $\int_{-2}^{5} g(x)\,dx = 10.$

Determine the value of $\int_{-2}^{2} (2 - g(x))\,dx + \int_{2}^{5} (2x - g(x))\,dx.$ (4 marks)



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**Question 16** **(8 marks)**

The concentration of a drug in the plasma of a monkey, $C$ micrograms per litre, $t$ hours after being administered, can be modelled by $C = C_0 e^{kt}$, where $C_0$ and $k$ are constants. Each dose of the drug increases the existing concentration by $430\ \mu g/L$, and the concentration of the drug is known to halve every $2$ hours and $40$ minutes.

A monkey, with no existing trace of the drug, was administered a first dose at $8{:}05$ am.

(a) Use the model to determine the rate of change of concentration of the drug in the monkey's plasma later that morning at $10{:}45$ am. (4 marks)

An additional dose is administered every time the concentration falls to $130\ \mu g/L$.

(b) Determine the expected time of day, to the nearest minute, that the third dose will be administered to the monkey. (4 marks)





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**Question 17** **(10 marks)**

A machine learning model is being developed to recognise a pathogen in medical images. The performance of the model is stable and the results of the last $250$ runs of the machine are shown in the table below.

| | | Model recognises a pathogen in image | |
|---|---|---|---|
| | | Yes | No |
| Image contains a pathogen | Yes | $154$ | $21$ |
| | No | $6$ | $69$ |

(a) Determine the probability that the model recognises a pathogen in a randomly selected image that contains a pathogen. (1 mark)

(b) The model is used to check $9$ randomly selected images. Determine the probability that it returns exactly one incorrect result. (3 marks)

(c) The model is used to check $30$ randomly selected images. Determine the probability that it returns at least $26$ correct results. (3 marks)

(d) The model is repeatedly used to check batches of $50$ randomly selected images that do not contain a pathogen. Determine the mean and standard deviation of the probability distribution for the number of correct results the model produces. (3 marks)





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**Question 18** **(10 marks)**

A small body moves in a straight line with velocity $v$ cm/s at time $t$ s given by

$$v(t) = 14 + 5\sin\left(\frac{\pi t}{6}\right) - 8\sin\left(\frac{\pi t}{3}\right), \qquad t \geq 0.$$

(a) By viewing the graph of the velocity function on your calculator, or otherwise, state the minimum velocity of the body for $t \geq 0$ to the nearest $0.01$ cm/s, and hence explain why the distance travelled by the body in any interval of time will always be the same as the change in displacement of the body. (2 marks)

(b) Determine the distance travelled by the body between $t = 0$ and $t = 12$. (2 marks)

The distance travelled ($x$ cm) by the body in any $6$ second interval from $t = T$ to $t = T + 6$ is given by the function $x(T) = a + b\cos\left(\frac{\pi T}{6}\right)$.

(c) Determine the value of the constant $a$ and the value of the constant $b$. (2 marks)

(d) During the first $20$ seconds, there is an $6$ second interval in which the distance travelled by the body is a minimum. Using calculus methods, determine when this interval occurs and justify that the distance is a minimum. (4 marks)





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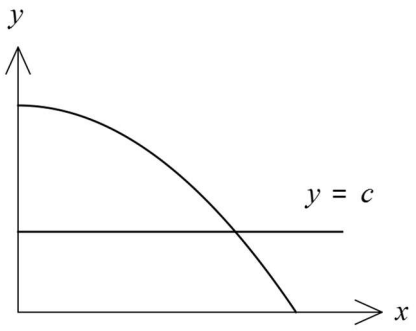
**Question 19** **(7 marks)**

The line $y = c$ divides the area in the first quadrant under the curve $y = 16 - x^2$ into two equal halves, as shown in the diagram.

Determine, with reasoning, the value of $c$.






**End of questions**

Supplementary page

Question number: _____